Inter (Part-I) 2021

Mathematics	Group-I	PAPER: I
Time: 2.30 Hours	(SUBJECTIVE TYPE)	Marks: 80

SECTION-I

2. Write short answers to any EIGHT (8) questions: **(16)**

(i) Prove that $\frac{a}{b} = \frac{ka}{kb}, k \neq 0$

Ans

$$\frac{a}{b} = \frac{a}{b} \cdot 1$$

$$= \frac{a}{b} \cdot \frac{k}{k} = \frac{ak}{bk}$$

$$\frac{a}{b} = \frac{ka}{kb} \text{ Proved.}$$

(ii) Simplify $(5, -4) + (-3, -8)$ and write the answer as a complex number.

Ans $(5, -4) + (-3, -8) = (5 - 4i) + (-3 - 8i)$

$$= \frac{(5 - 4i)}{(-3 - 8i)} \times \frac{(-3 + 8i)}{(-3 + 8i)} = \frac{(5 - 4i)(-3 + 8i)}{(-3)^2 - (8i)^2}$$

$$= \frac{5(-3 + 8i) - 4i(-3 + 8i)}{9 - 64i^2}$$

$$= \frac{-15 + 40i + 12i - 32i^2}{9 - 64(-1)}$$

$$= \frac{-15 + 52i - 32(-1)}{9 + 24}$$

$$= \frac{-15 + 52i + 32}{73} = \frac{17 + 52i}{73}$$

$$= \frac{17}{73} + \frac{52}{73} i$$

(iii) Find the real and imaginary parts of $(\sqrt{3} + i)^3$.

Ans Let $\quad r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$ where

$r^2 = (\sqrt{3})^2 + 1^2$ or $r = \sqrt{3 + 1} = 2$ and $\theta = \tan^{-1}\frac{1}{\sqrt{3}} = 30^\circ$

So, $\quad (\sqrt{3} + i)^3 = (r\cos\theta + i\, r\sin\theta)^3$

$= r^3 (\cos 3\theta + i \sin 3\theta)$ (By De Moivre's Theorem)

$= 2^3 (\cos 90^\circ + i \sin 90^\circ)$

$= 8(0 + i \cdot 1) = 8i$

Thus, 0 and 8 are respectively real and imaginary parts of $(\sqrt{3} + i)^3$.

(iv) If B = {1, 2, 3}, then find the power set of B, *i.e.*, P(B).

Ans If B = {1, 2, 3}, then

$P(B) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Construct the truth table for the statement:

$$\sim (p \rightarrow q) \leftrightarrow (p \wedge \sim q)$$

Ans

p	q	$\sim q$	$p \rightarrow q$	$\sim (p \rightarrow q)$	$p \wedge \sim q$	$\sim (p \rightarrow q) \rightarrow (p \wedge \sim q)$	$(p \wedge \sim q) \rightarrow \sim (p \rightarrow q)$
T	T	F	T	F	F	T	T
T	F	T	F	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	F	F	T	T

(vi) For the set A = {1, 2, 3, 4}, find a relation in A which satisfy {(x, y) | y + x = 5}.

Ans Relation in A:

$r = \{(x, y) \mid y + x = 5\}$

$= \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

(vii) Find the matrix X, if 2X – 3A = B and

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

Ans $2X - 3A = B$

$2X = B + 3A$

$$2X = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & -1 \end{bmatrix} + 3\begin{bmatrix} 1 & -1 & 2 \\ -2 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -3 & 6 \\ -6 & 12 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 3+3 & -1-3 & 0+6 \\ 4-6 & 2+12 & 1+15 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 6 & -4 & 6 \\ -2 & 14 & 16 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{2} & \frac{-4}{2} & \frac{6}{2} \\ \frac{-2}{2} & \frac{14}{2} & \frac{16}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} 3 & -2 & 3 \\ -1 & 7 & 8 \end{bmatrix}$$

(viii) Find A^{-1} if $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$.

Ans $|A| = \begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix} = 5 - 3 = 2$

Since $|A| \neq 0$, we can find A^{-1}.

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\Rightarrow \quad A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-3}{2} \\ \frac{-1}{2} & \frac{5}{2} \end{bmatrix}$$

(ix) Without expansion, show that $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$.

Ans $\text{L.H.S} = \begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix}$

By $(c_1 + c_2)$,

$$= \begin{vmatrix} \alpha+\beta+\gamma & \beta+\gamma & 1 \\ \alpha+\beta+\gamma & \gamma+\alpha & 1 \\ \alpha+\beta+\gamma & \alpha+\beta & 1 \end{vmatrix}$$

Taking common $(\alpha + \beta + \gamma)$ from c_1,

$$= (\alpha+\beta+\gamma)\begin{vmatrix} 1 & \beta+\gamma & 1 \\ 1 & \gamma+\alpha & 1 \\ 1 & \alpha+\beta & 1 \end{vmatrix}$$

$$= (\alpha+\beta+\gamma)(0) = 0 = \text{R.H.S.}$$

(x) Prove that sum of cube roots of unity is zero *i.e.,* $1 + \omega + \omega^2 = 0$.

Ans We know that cube roots of unity are:

$$1, \frac{-1+\sqrt{3}\,i}{2} \text{ and } \frac{-1-\sqrt{3}\,i}{2}$$

If $\omega = \dfrac{-1+\sqrt{3}\,i}{2}$,

then $\omega^2 = \dfrac{-1-\sqrt{3}\,i}{2}$.

Sum of all the three cube roots,

$$1 + \omega + \omega^2 = 1 + \frac{-1+\sqrt{3}\,i}{2} + \frac{-1-\sqrt{3}\,i}{2}$$

$$= \frac{2-1+\sqrt{3}\,i-1-\sqrt{3}\,i}{2}$$

$$= \frac{0}{2} = 0$$

Hence sum of cube roots of unity

$1 + \omega + \omega^2 = 0$

(xi) Find the numerical value of k, when the polynomial $x^3 + kx^2 - 7x + 6$ has a remainder of -4 when divided by $x + 2$.

Ans Let $f(x) = x^3 + kx^2 - 7x + 6$

and $x - a = x + 2$, we have

$a = -2$

(By Remainder Theorem)

Remainder $= f(-2)$

$= (-2)^3 + k(-2)^2 - 7(-2) + 6$

$= -8 + 4k + 14 + 6$

$= 4k + 12$

Given that remainder $= -4$

$\therefore \quad 4k + 12 = -4$

$4k = -4 - 12$

$4k = -16$

$k = -4$

(xii) Show that the roots of equation $x^2 + (mx + c)^2 = a^2$ will be equal if $c^2 = a^2(1 + m^2)$.

Ans $x^2 + (mx + c)^2 = a^2$

$\Rightarrow \quad x^2 + m^2x^2 + 2mcx + c^2 = a^2$

$(1 + m^2)x^2 + (2mc)x + (c^2 - a^2) = 0$

Here, $a = 1 + m^2$, $b = 2mc$, $c = c^2 - a^2$

Now,

$D = \text{Discriminant} = b^2 - 4ac$

$= 4m^2c^2 - 4(1 + m^2)(c^2 - a^2)$

$$= 4[m^2c^2 - (m^2c^2 - m^2a^2 + c^2 - a^2)]$$
$$= 4[m^2c^2 - m^2c^2 + m^2a^2 - c^2 + a^2]$$
$$= 4[m^2a^2 - c^2 + a^2]$$
$$D = 4[a^2(1 + m^2) - c^2]$$

Now roots are equal if $D = 0$

So, $4[a^2(1 + m^2) - c^2] = 0$

$\Rightarrow$ $a^2(1 + m^2) - c^2 = 0$

$a^2(1 + m^2) = c^2$

$\Rightarrow$ $\boxed{c^2 = a^2(1 + m^2)}$ Proved.

3. Write short answers to any EIGHT (8) questions: (16)

(i) Resolve $\dfrac{4x^2}{(x^2+1)^2(x-1)}$ into partial fractions.

Ans Let $\dfrac{4x^2}{(x^2+1)^2(x-1)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{(x^2+1)^2} + \dfrac{E}{x-1}$

$\Rightarrow 4x^2 = (Ax + B)(x^2 + 1)(x - 1) + (Cx + D)(x - 1) + E(x^2 + 1)^2$ (i)

$\Rightarrow 4x^2 = (A + E)x^4 + (-A + B)x^3 + (A - B + C + 2E)x^2$
$+ (-A + B - C + D)x + (-B - D + E)$ (ii)

Putting $x - 1 = 0 \Rightarrow x = 1$ in (i), we get

$$4 = E(1 + 1)^2 \Rightarrow \boxed{E = 1}$$

Equating the coefficients of x^4, x^3, x^2, x, in (ii), we get

$$0 = A + E \Rightarrow A = -E \Rightarrow \boxed{A = -1}$$
$$0 = -A + B \Rightarrow B = A \Rightarrow \boxed{B = -1}$$
$$4 = A - B + C + 2E$$
$$\Rightarrow C = 4 - A + B - 2E = 4 + 1 - 1 - 2 \Rightarrow \boxed{C = 2}$$
$$0 = -A + B - C + D$$
$$\Rightarrow D = A - B + C = -1 + 1 + 2 = 2 \Rightarrow \boxed{D = 2}$$

Hence, partial fractions are: $\dfrac{-x-1}{x^2+1} + \dfrac{2x+2}{(x^2+1)^2} + \dfrac{1}{x-1}$

(ii) Resolve $\dfrac{7x+25}{(x+3)(x+4)}$ into partial fractions.

Ans Suppose $\dfrac{7x+25}{(x+3)(x+4)} = \dfrac{A}{x+3} + \dfrac{B}{x+4}$

$\Rightarrow$ $7x + 25 = A(x + 4) + B(x + 3)$

As two sides of the identity are equal for all values of x, let us put $x = -3$, and $x = -4$ in it.

Putting $x = -3$, we get $-21 + 25 = A(-3 + 4)$

$$\Rightarrow \boxed{A = 4}$$

Putting $x = -4$, we get $-28 + 25 = B(-4 + 3)$

$$\Rightarrow \boxed{B = 3}$$

Hence, the partial fractions are: $\dfrac{4}{x+3} + \dfrac{3}{x+4}$.

(iii) Write the first four terms of the sequence, $a_n = (-1)^n n^2$.

Ans Given: $a_n = (-1)^n n^2$

By putting n = 1, 2, 3, 4 in the above sequence, we have

$$a_1 = (-1)^1 (1)^2 \Rightarrow (-1)(1) = -1$$
$$a_2 = (-1)^2 (2)^2 \Rightarrow 1(4) = 4$$
$$a_3 = (-1)^3 (3)^2 \Rightarrow (-1)(9) = -9$$
$$a_4 = (-1)^4 (4)^2 \Rightarrow (1)(16) = 16$$

So, the first four terms of the sequence are $-1, 4, -9, -16$.

(iv) If $a_{n-3} = 2n - 5$, find nth term of the sequence.

Ans Here, $a_{n-3} = 2n - 5$

By putting, n = 4, 5, 6, and 7.

For n = 4,

$$a_{4-3} = 2(4) - 5$$
$$a_1 = 8 - 5$$
$$\boxed{a_1 = 3}$$

For n = 5,

$$a_{5-3} = 2(5) - 5$$
$$a_2 = 10 - 5$$
$$a_2 = 5$$

For n = 6,

$$a_{6-3} = 2(6) - 5 = 12 - 5$$
$$a_3 = 7$$

For n = 7,

$$a_{7-3} = 2(7) - 5 = 14 - 5$$
$$a_4 = 9$$

Thus $a_1 = 3, a_2 = 5, a_3 = 7, a_4 = 9$ is an average progression (A.P).
The common difference

$$d = a_2 - a_1 = a_3 - a_2 = 2$$

$$a_n = a + (n-1)d$$

$$= 3 + (n-1)(2) = 3 + 2n - 2$$

$$a_n = 2n + 1$$

(v) Insert two G.M's between 2 and 16.

Ans Let G_1, G_2 be the two G.Ms between 2 and 16.

Then $2, G_1, G_2, 16$ are in G.P.

Here, $a = 2, n = 4, a_4 = 16$

We know $a_n = a\,r^{n-1}$

For $n = 4$, $a_4 = a\,r^{4-1}$

$\Rightarrow$ $16 = 2(r)^3$

$\Rightarrow$ $r^3 = \dfrac{16}{2} = 8$

$\Rightarrow$ $r^3 = (2)^3$

$\Rightarrow$ $r = 2$

Thus $G_1 = ar = 2(2) = 4$

$G_2 = ar^2 = 2(2)^2 = 8$

Thus the two G.M's between 2 and 16 are 4, 8.

(vi) Sum the infinite geometric series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \text{------}$

Ans Here $a = \dfrac{1}{2}$, $a_1 = \dfrac{1}{4}$

$$r = \frac{a_1}{a} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{4} \cdot 2 = \frac{1}{2}$$

Sum of infinite geometric series $= s_\infty = \dfrac{a}{1-r}$

$$= \frac{\left(\frac{1}{2}\right)}{1 - \frac{1}{2}}$$

$$= \frac{\frac{1}{2}}{\frac{2-1}{2}} = \frac{1}{2} \cdot \frac{2}{1}$$

$$S_\infty = 1$$

(vii) Find the value of n, when $^{11}P_n = 11.10.9$.

Ans $$^{11}P_n = \frac{11!}{(11-n)!} = 11 \times 10 \times 9$$

$$\Rightarrow \quad \frac{11!}{(11-n)!} = 990$$

$$\frac{(11-n)!}{11!} = \frac{1}{990}$$

$$(11-n)! = \frac{11!}{990}$$

$$(11-n)! = \frac{11 \times 10 \times 9 \times 8!}{11 \times 10 \times 9}$$

$$(11-n)! = 8!$$

$$11 - n = 8$$

$$\Rightarrow \quad n = 11 - 8$$

$$n = 3$$

(viii) Evaluate $^{12}C_3$.

Ans $$^nC_r = \frac{n!}{(n-r)!\,r!} \quad \text{(i)}$$

By putting $n = 12$, $r = 3$ in (i), we have

$$^{12}C_3 = \frac{12!}{(12-3)!\,3!}$$

$$= \frac{12!}{9!\,3!}$$

$$= \frac{12 \times 11 \times 10 \times 9!}{9!\;3 \times 2 \times 1} = 4 \times 11 \times 5$$

$$= 220$$

(ix) A die is rolled. What is the probability that the dots on the top are greater than 4?

Ans $$S = \{1, 2, 3, 4, 5, 6\}$$

$\Rightarrow$ $n(S) = 6$

The event E that the dots on the top are greater than 4 = {5, 6}

$\Rightarrow$ $n(E) = 2$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

(x) Check the truth of the statement $1 + 5 + 9 + \text{-----} + (4n - 3) = n(2n - 1)$ for $n = 1, 2$.

Ans Let S(n) be the given statement, *i.e.*,

$S(n) : 1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1) \quad \ldots (i)$

If n = 1, then S(n) = S(1) becomes

$1(2(1) - 1) \Rightarrow 1(2 - 1) = 1$

which is true.

Thus, the condition (1) is satisfied as S(1) is true.

Let's assume S(n) is true for $n = k \in N$, *i.e.*,

$1 + 5 + 9 + \ldots\ldots + (4k - 3) = k(2k - 1) \quad \ldots (ii)$

By adding (4k + 1) in both sides, we have

$$\begin{aligned} 1 + 5 + 9 + \ldots + (4k - 3) + (4k + 1) &= k(2k - 1) + (4k + 1) \\ &= 2k^2 + 3k + 1 \\ &= 2k^2 + 2k + k + 1 \\ &= 2(k + 1) + 1(k + 1) \\ &= (k + 1)(2k + 1) \end{aligned}$$

or $\quad 1 + 5 + 9 + \ldots + (4k + 4 - 3) = (k + 1)(2k + 2 - 1)$

$1 + 5 + 9 + \ldots + [4(k + 1) - 3] = (k + 1)[2(k + 1) - 1] \quad \ldots (iii)$

The equation (iii) is of same from as the equation (ii) except that k is replaced by k + 1.

Thus from S(k), we have obtained S(k + 1), so the condition (2) is satisfied. Since both the conditions are satisfied, so the statement $1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$ is true for n = 1, 2.

(xi) Calculate by means of binomial theorem $(2.02)^4$.

Ans $(2.02)^4 = (2 + 0.02)^4$

$$= 2^4 + \binom{4}{1} 2^3 (0.02) + \binom{4}{2} 2^2 (0.02) + \binom{4}{3} 2 (0.02)^3 + \binom{4}{4} (0.02)^4$$

$$= 16 + (4)(8)(0.02) + \left(\frac{4 \times 3}{2 \times 1}\right)(4)(0.004) + (4)(2)(0.000008) + 1(0.00000016)$$

$$= 16 + 0.64 + 0.0096 + 0.000064 + 0.00000016$$

$$= 16.64966416$$

(xii) If x is so small that its square and higher powers can be neglected, then show that $\frac{\sqrt{1+2x}}{\sqrt{1-x}} \approx 1 + \frac{3}{2}x$.

Ans $L.H.S = \frac{\sqrt{1+2x}}{\sqrt{1-x}} = (1+2x)^{1/2}(1-x)^{-1/2}$ (i)

Take $(1+2x)^{1/2} = \left\{1 + \frac{1}{2}(2x) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}(2x)^2 + \right\}$

$= \{1 + x\}$ neglecting x^2 and higher powers of x.

Now, $(1-x)^{-1/2} = \left\{1 + \left(-\frac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}(-x)^2 + \ldots\right\}$

$= \left\{1 + \frac{x}{2}\right\}$ neglecting x^2 and higher powers of x.

Putting in eq. (i), we get

$L.H.S = \{1 + x\}\{1 + \frac{x}{2}\}$

$= 1 + \frac{x}{2} + x + \frac{x^2}{2} = 1 + \frac{x+2x}{2}$ neglecting x^2

$= 1 + \frac{3x}{2} \cong R.H.S.$

4. Write short answers to any NINE (9) questions: (18)

(i) Convert 54° 45′ into radians.

Ans $54^{\circ}45' = \left[54 + \frac{45}{60}\right]^{\circ} = \left[54 + \frac{3}{4}\right]^{\circ} = \frac{219^{\circ}}{4}$

$= \frac{219\,(1^{\circ})}{4}$

$\approx \frac{219}{4}(0.0175)$ radians

≈ 0.958 radians.

(ii) If $\cot\theta = \frac{15}{8}$ and the terminal arm of the angle is not in quadrant I, find the value of cosec θ.

Ans Given that the terminal arm of the angle is not in quadrant I. We know that cot θ is positive in the quadrant I and

it is also positive in the quadrant III. Therefore, the terminal arm of the angle is in the quadrant III.

$$\operatorname{cosec}^2 \theta = 1 + \cot^2 \theta$$

$$= 1 + \left(\frac{15}{8}\right)^2$$

$$= 1 + \frac{225}{64} = \frac{289}{64}$$

$$\operatorname{cosec} \theta = \pm \frac{17}{8}$$

Since the terminal arm of the angle is in the III quadrant where cosec θ is negative.

$$\therefore \quad \operatorname{cosec} \theta = \frac{-17}{8}$$

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta} = \frac{-8}{17}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\cos \theta = \cot \theta \sin \theta$$

$$= \frac{15}{8} \times \frac{-8}{17} = \frac{-15}{17}$$

(iii) Verify $2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

Ans L.H.S $= 2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ$

$$= 2\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2}(\sqrt{2})$$

$$= (\sqrt{2})(\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) + \frac{1}{(\sqrt{2})(\sqrt{2})}(\sqrt{2})$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}$$

$$= \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \text{R.H.S}$$

(iv) Prove that $\cot(\alpha + \beta) = \frac{\cot \alpha \cot \beta - 1}{\cot \alpha + \cot \beta}$.

Ans L.H.S $= \cot(\alpha + \beta) = \frac{1}{\tan(\alpha + \beta)}$

$$= \frac{1}{\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$$

$$= \frac{1 - \tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$$

$$= \frac{1 - \frac{1}{\cot\alpha}\frac{1}{\cot\beta}}{\frac{1}{\cot\alpha} + \frac{1}{\cot\beta}}$$

$$= \frac{\frac{\cot\alpha\cot\beta - 1}{\cot\alpha\cot\beta}}{\frac{\cot\beta + \cot\alpha}{\cot\alpha\cot\beta}}$$

$$= \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$$

= R.H.S Proved.

(v) Prove that $\tan(180^\circ + \theta) = \tan\theta$.

Ans L.H.S = $\tan(180^\circ + \theta)$

$$= \frac{\tan 180^\circ + \tan\theta}{1 - \tan 180^\circ \tan\theta}$$

$$= \frac{0 + \tan\theta}{1 - 0(\tan\theta)} = \tan\theta = \text{R.H.S.}$$

(vi) Express $2\sin 7\theta \sin 2\theta$ as sums or differences.

Ans $2\sin 7\theta \sin 2\theta = \cos(7\theta - 2\theta) - \cos(7\theta + 2\theta)$

$= \cos 5\theta - \cos 9\theta$

(vii) Find the period of $\tan\frac{x}{7}$.

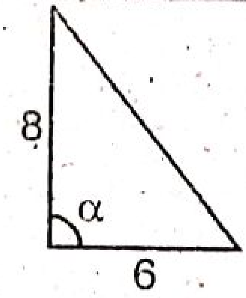
Ans $\tan\frac{x}{7} = \tan\frac{1}{7}[x + 7(\pi)]$

$= \tan(x + \pi)$

$= \tan x$

So, the period of $\tan\frac{x}{7}$ is 7π.

(viii) A vertical pole is 8 m high and the length of its shadow is 6 m. What is the angle of elevation of the Sun at that moment?

Ans 

$$\tan\alpha = \frac{8}{6}$$
$$= \frac{4}{3}$$
$$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$
$$\alpha = 53^\circ\, 7'$$

(ix) Find area of the triangle ABC if $a = 200$, $b = 120$, $\gamma = 150^\circ$.

Ans Area of triangle $\Delta = \frac{1}{2}ab\sin\gamma$

$$= \frac{1}{2}(200)(120)\sin 150^\circ$$
$$= \frac{1}{2}(200)(120)(0.5)$$
$$\Delta = 6000 \text{ sq units}$$

(x). Prove that $r\,r_1\,r_2\,r_3 = \Delta^2$.

Ans $$r\,r_1\,r_2\,r_3 = \frac{\Delta}{s}\times\frac{\Delta}{s-a}\times\frac{\Delta}{s-b}\times\frac{\Delta}{s-c}$$
$$= \frac{\Delta^4}{s(s-a)(s-b)(s-c)}$$
$$= \frac{\Delta^4}{\Delta^2}$$
$$= \Delta^2 \quad \text{Proved.}$$

(xi) Find the value of $\sec\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$

Ans We first find the value of y, whose sine is $-\frac{1}{2}$.

$$\sin y = -\frac{1}{2}, \qquad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$
$$\Rightarrow \quad y = -\frac{\pi}{6}$$

$$\Rightarrow \quad \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \quad \sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right] = \frac{2}{\sqrt{3}}$$

(xii) **Show that** $r = (s-a)\tan\left(\frac{\alpha}{2}\right)$.

Ans To prove $r = (s-a)\tan\frac{\alpha}{2}$

We know that: $\tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

$$\text{R.H.S.} = (s-a)\tan\frac{\alpha}{2} = (s-a)\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

$$= \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

$$= \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s^2} = \frac{\Delta}{s} = r$$

$$\therefore \quad (s-a)\tan\frac{\alpha}{2} = r$$

(xiii) **Find the solution of cosec $\theta = 2$ which lies in the interval $[0, 2\pi]$.**

Ans

$$\text{cosec}\,\theta = 2$$

or $$\frac{1}{\text{cosec}\,\theta} = \frac{1}{2}$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2}$$

$\therefore$ $\sin\theta$ is positive in first and second quadrants with the angle $\theta = \frac{\pi}{6}$.

$$\therefore \quad \theta = \frac{\pi}{6}$$

and $$\theta = \pi - \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6} \qquad \frac{\pi}{6}, \frac{5\pi}{6}$$

SECTION-II

NOTE: Attempt any Three (3) questions.

Q.5.(a) Solve by Cramer's rule $2x_1 - x_2 + x_3 = 8$, $x_1 + 2x_2 + 2x_3 = 6$, $x_1 - 2x_2 - x_3 = 1$ **(5)**

Ans From the above system of linear equations:

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 2 \\ 1 & -2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 1 \end{bmatrix}$$

$$A \quad X = B$$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 2 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= 2(-2+4) - (-1)(-1-2) + 1(-2-2)$$

$$= 2(2) + (-3) + (-4) = 4 - 3 - 4 = -3$$

$$|A_1| = \begin{vmatrix} 8 & -1 & 1 \\ 6 & 2 & 2 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= 8(-2+4) - (-1)(-6-2) + 1(-12-2)$$

$$= 8(2) + (-8) + (-14) = 16 - 8 - 14 = -6$$

$$|A_2| = \begin{vmatrix} 2 & 8 & 1 \\ 1 & 6 & 2 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 2(-6-2) - 8(-1-2) + 1(1-6)$$

$$= 2(-8) - 8(-3) + (-5)$$

$$= -16 + 24 - 5 = 3$$

$$|A_3| = \begin{vmatrix} 2 & -1 & 8 \\ 1 & 2 & 6 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= 2(2+12) - (-1)(1-6) + 8(-2-2)$$

$$= 2(14) + (-5) + 8(-4)$$

$$= 28 - 5 - 32 = -9$$

$$x_1 = \frac{|A_1|}{|A|} \Rightarrow \frac{-6}{-3} = 2$$

$$x_2 = \frac{|A_2|}{|A|} \Rightarrow \frac{3}{-3} = -1$$

$$= \frac{c}{a} + \left(\frac{b^2 - 2ac}{a^2}\right)\left(\frac{a}{c}\right) + \frac{a}{c}$$

$$= \frac{c}{a} + \frac{b^2 - 2ac}{ac} + \frac{a}{c}$$

$$= \frac{c^2 + b^2 - 2ac + a^2}{ac} = \frac{a^2 + b^2 + c^2 - 2ac}{ac}$$

So, the required equation can be:

$$x^2 - Sx + P = 0$$

$$x^2 - \left[-\frac{b(c+a)}{ac}\right]x + \left(\frac{a^2 + b^2 + c^2 - 2ac}{ac}\right) = 0$$

By simplifying, we get

$$acx^2 + b(c+a)x + (a^2 + b^2 + c^2 - 2ac) = 0$$

Q.6.(a) Resolve $\frac{3x - 11}{(x^2 + 1)(x + 3)}$ into partial fraction. (5)

Ans Suppose $\frac{3x - 11}{(x^2 + 1)(x + 3)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 3}$

$\Rightarrow \quad 3x - 11 = (Ax + B)(x + 3) + C(x^2 + 1) \qquad$ (i)

$\Rightarrow \quad 3x - 11 = (A + C)x^2 + (3A + B)x + (3B + C) \qquad$ (ii)

Putting $x + 3 = 0 \quad \Rightarrow \quad x = -3$ in (i), we get

$$-9 - 11 = C(9 + 1) \Rightarrow \boxed{C = -2}$$

Equating the coefficients of x^2 and x in (ii), we get

$$0 = A + C \Rightarrow A = -C \Rightarrow \boxed{A = 2}$$

and $\quad 3 = 3A + B \Rightarrow B = 3 - 3A \Rightarrow B = 3 - 6$

$$\Rightarrow \boxed{B = -3}$$

Hence, the partial fraction are: $\frac{2x - 3}{x^2 + 1} - \frac{2}{x + 3}$

(b) If $S_n = n(2n - 1)$, then find the series. (5)

Ans Given that $S_n = n(2n - 1) \qquad \ldots$ (i)

Putting $n = 1, 2, 3, 4$ in (i), we get

$S_1 = 1(2 \times 1 - 1) = 2 - 1 = 1;$

$S_2 = 2(2 \times 2 - 1) = 2 \times 3 = 6;$

$S_3 = 3(2 \times 3 - 1) = 3 \times 5 = 15;$

$S_4 = 4(2 \times 4 - 1) = 4 \times 7 = 28$

Now $a_1 = S_1 = 1,\ a_2 = S_2 - S_1 = 6 - 1 = 5$

$$a_3 = S_3 - S_2 = 15 - 6 = 9$$

and $a_4 = S_4 - S_3 = 28 - 15 = 13$

Thus the required series is $1 + 5 + 9 + 13 + ...$

Q.7.(a) Prove that $^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$. **(5)**

Ans For Answer See Paper 2019 (Group-II), Q.6.(b).

(b) Use mathematical induction to prove **(5)**

$$\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + ... + \binom{n+2}{3} = \binom{n+3}{4}$$

for every positive integers n.

Sol Let S(n) be the given statement, *i.e.,*

$$S(n) : 1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(4n^2-1)}{3} \quad ... (i)$$

If we put n = 1, then (i) becomes

$$1^2 = \frac{1[4(1)^2 - 1]}{3}$$

$$= \frac{4-1}{3} = \frac{3}{3} = 1 \text{ True}$$

Thus, the condition (I) is satisfied as S(1) is true.

If we assume that S(n) is true for $n = k \in N$, *i.e.,*

$$1^2 + 3^2 + 5^2 + ... + (2k-1)^2 = \frac{k(4k^2-1)}{3} \quad ... (ii)$$

By adding $(2k+1)^2$ on both sides, we get

$$1^2 + 3^2 + 5^2 + ... + (2k-1)^2 + (2k+1)^2 = \frac{k(4k^2-1)}{3} + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$$

$$= \frac{2k+1}{3}[k(2k-1) + 3(2k+1)]$$

$$= \frac{2k+1}{3}[2k^2 + 5k + 3]$$

$$= \frac{2k+1}{3}(k+1)(2k+3)$$

$$= \frac{k+1}{3}[(2k+1)(2k+3)]$$

$$= \frac{k+1}{3}[4k^2 + 8k + 3]$$

$$= \frac{k+1}{3}[4(k^2 + 2k + 1) - 11]$$

$$1^2 + 3^2 + 5^2 + \ldots + (2(k+1) - 1)^2 = \frac{(k+1)[4(k+1)^2 - 1]}{3} \quad \text{(iii)}$$

Equation (iii) is true as condition (2) is satisfied. Since both the conditions are satisfied, so the given statement (1) is true for every positive integer n.

Q.8.(a) Two cities A and B lies on the equator, such that their longitudes are 45° E and 25° W, respectively. Find the distance between the two cities, taking the radius of the Earth as 6,400 kms. (5)

Ans

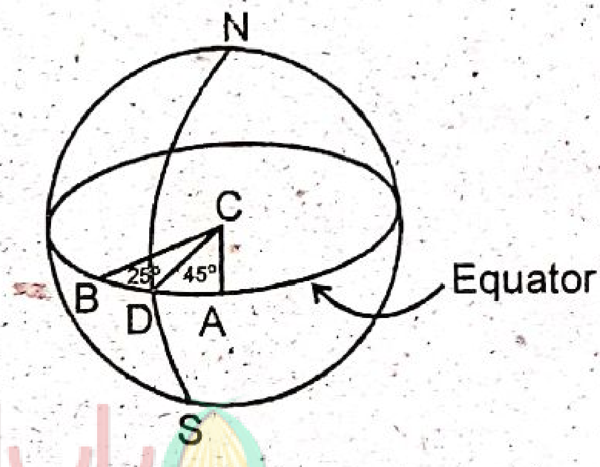
$r = 6400$ km

$\theta = 45^\circ + 25^\circ$

$= 70^\circ$

$= 70 \times \frac{\pi}{180}$

$= \frac{7}{18}\pi$ radians



Let ℓ be the arc AB, therefore

$\ell = r\theta$

$= 6400\left(\frac{7}{18}\pi\right)$

$\simeq 6400\,(0.3889 \times 3.14159)$

$\simeq 6400\,(1.22176)$

$= 7819.264$

Thus, the distance between two cities is

$\ell = 7819$ k (approximately).

(b) Prove that $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$. (5)

Ans For Answer see Paper 2017 (Group-II), Q.8.(b).

Q.9.(a) Solve the triangle ABC, if a = 53, $\beta = 88^\circ 36'$, $\gamma = 31^\circ 54'$. (5)

Ans $\alpha + \beta + \gamma = 180^\circ$

$$\alpha = 180^\circ - \beta - \gamma$$
$$\alpha = 180 - 88^\circ\, 36'\, 31^\circ\, 54'$$
$$\alpha = 59^\circ\, 30'$$

By using law of sine, we have

$$\frac{c}{\sin v} = \frac{a}{\sin \alpha}$$

$$\Rightarrow \quad c = \frac{a \sin \gamma}{\sin \alpha}$$

$$= \frac{53 \sin 31^\circ\, 54'}{\sin 59^\circ\, 30'}$$

$$c = \frac{53\,(0.5284)}{0.8616} = \frac{28.0052}{0.8616}$$

$$\boxed{c = 32.5}$$

And $\frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$

$$\Rightarrow \quad b = \frac{c \sin \beta}{\sin \gamma}$$

$$= \frac{32.5 \sin 88^\circ\, 36'}{\sin 31^\circ\, 54'} = \frac{(32.5)\,(0.9997)}{0.5284} = \frac{32.4903}{0.5284}$$

$$\boxed{b = 61.5}$$

So, the triangle ABC have

$a = 53$, $b = 61.5$, $c = 32.5$

$\alpha = 59^\circ\, 30'$, $\beta = 88^\circ\, 36'$, $\gamma = 31^\circ\, 54'$

(b) Prove that $\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$. **(5)**

Ans By using $\tan^{-1} a + \tan^{-1} b = \tan^{-1}\frac{a+b}{1-ab}$

$$\text{L.H.S} = \tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\left(\frac{\frac{1}{11} + \frac{5}{6}}{1 - \frac{1}{11} \times \frac{5}{6}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6+55}{66}}{\frac{66-5}{66}}\right)$$

$$= \tan^{-1}\left(\frac{61}{61}\right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \frac{\pi}{4} \qquad \text{... (i)}$$

Next consider the right hand side

$$R.H.S = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$$

$$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}}\right)$$

$$= \tan^{-1}\left(\frac{5}{5}\right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2} = \frac{\pi}{4} \qquad \text{... (ii)}$$

Equating (i) and (ii), we have

$$\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \frac{\pi}{4} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$$

$$\Rightarrow \tan^{-1}\frac{1}{11} + \tan^{-1}\frac{5}{6} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$$